# Markets and the Environment

Exercises 1
Natural Resource Management

2014/2015

1. Assume that the (inverse) demand for a non-renewable resource $Z$ is given by the function $p_t = 1000 - 5\,Z_t$ and that the interest rate is $r = 5\%$ and $p_0 = 100$. Calculate:
   a) The optimal extraction period $T$ of the resource, the total quantity extracted during this period, the backstop price and the remaining resource stock.
   b) Holding the extraction period $T$ fixed, what would the interest rate have to be if $p_0$ changed to 200?
   c) What price $p_0$ do you obtain if the stock of the resource is 6000 units and $T$ and $r$ take the same values as in part a)
   d) Suppose now that the backstop price drops to $p_s = 800$. What is the new optimal extraction period $T'$ and how much of the stock has been extracted after 10 years with $r = 5\%$ and $p_0 = 100$?

2. Suppose that the world price of copper follows Hotelling's rule.
   a) Following the financial crisis, many central banks around the world reduced interest rates $r$. In a diagram, show the effects of this worldwide reduction in interest rates on the path of the copper price and on the optimal extraction period for cooper.
   b) After the change in the interest rate, it becomes known that global demand for copper will increase because of a surge in demand from emerging economies. In the same diagram, show how this affects the price path and the optimal extraction period.

3. The sustainable harvest of a renewable resource is given by the following growth function
$$F(X) = gX\left(1 - \frac{X}{X_max}\right)$$

For a given stock of the resource $X$, by exerting effort $E$ the following quantity can be harvested

$$H(X, E) = hEX$$

where $h$ is a constant. The price $p$ of the resource and the cost per unit of effort $c$ are constant.

a) Show that in the equilibrium with free entry and sustainable harvests the effort and stock are given by

$$E = \frac{g}{h}\left(1 - \frac{c}{phX_{max}}\right) \quad and \quad X = \frac{c}{ph}$$

b) How does the effort and the stock change if the cost-price ratio $\frac{c}{p}$ increases? In particular, what is the effect on $E$ when this ratio becomes very large? Explain the intuition behind your result.

4. The growth function of a renewable resource is given by

$$F(X) = 20X - 0,2X^2$$

where $X$ is the resource stock. Given a constant resource price $p = 10$ and constant private extraction cost $C = 4200$,

a) Derive the resource stock and the sustainable harvest under free entry?

b) Derive the resource stock and the sustainable harvest with a single owner. Calculate the profits.

c) Does any of the solutions in the previous paragraphs coincide with the Maximum Sustainable Yield (MSY)? Explain your answer.

d) Suppose now that resource extraction generates an external cost of $CE(X) = 8(100-X)$? Derive the optimal resource stock and calculate the social welfare associated with it.

5. Consider a recently planted forest. Suppose that the volume of commercial timber after $t$ periods is given by $X(t) = 10t + t^2 - 0,01t^3$

a) What is the maximum timber volume and in which period is it attained?

b) What rotation length maximizes the average product $X(t)/t$, and what is the corresponding quantity?

c) At a price of $p = 1$ and a discount rate of $r = 0.05$, verify that the optimal length of a single rotation equals $t^* = 28.1415$. Calculate the volume and the present value of the timber for this rotation length.